11 – Introduction to Gust / Turbulence Dynamic Response

Part 2: Continuous Turbulence response

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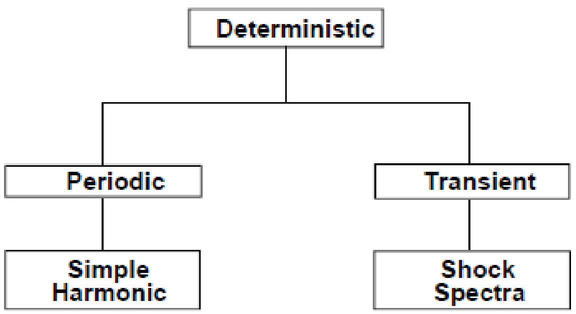
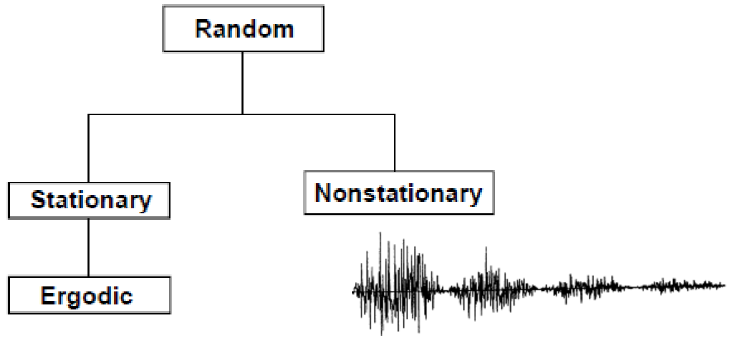
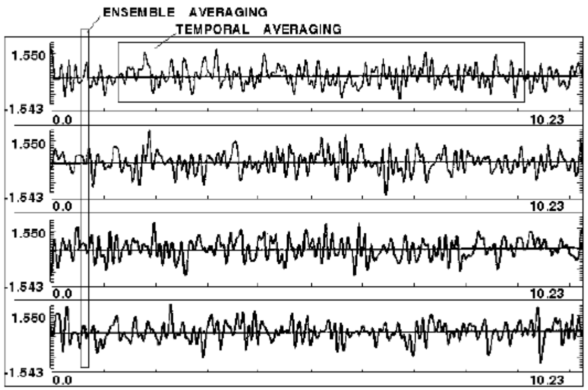






- Stationary random: the mean is constant and the nature of the signal remains the sam
- Non-stationary random: the mean may vary and the nature of the signal changes
- Ergodic random: a sample can be taken of the signal, or across a signal and it will be representative of the event

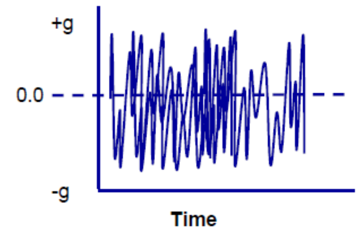
RANDOM SIGNALS: INTRODUCTION

- Random phenomenon can be described only in a statistical sense. Its instantaneous magnitude at any time is not known; rather, the probability of its magnitude exceeding a certain value is given

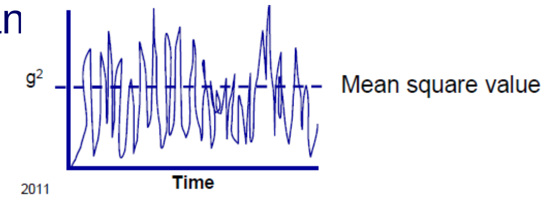
- Examples include earthquake ground motion, ocean wave heights and frequencies, wind pressure fluctuations on aircraft and tall buildings, and acoustic excitation due to rocket and jet engine noise

- Characterization of a random signal:
  - Root Mean Square
  - Cumulative Mean Square
  - Power Spectral Density

- Assume random signal with zero mean value:

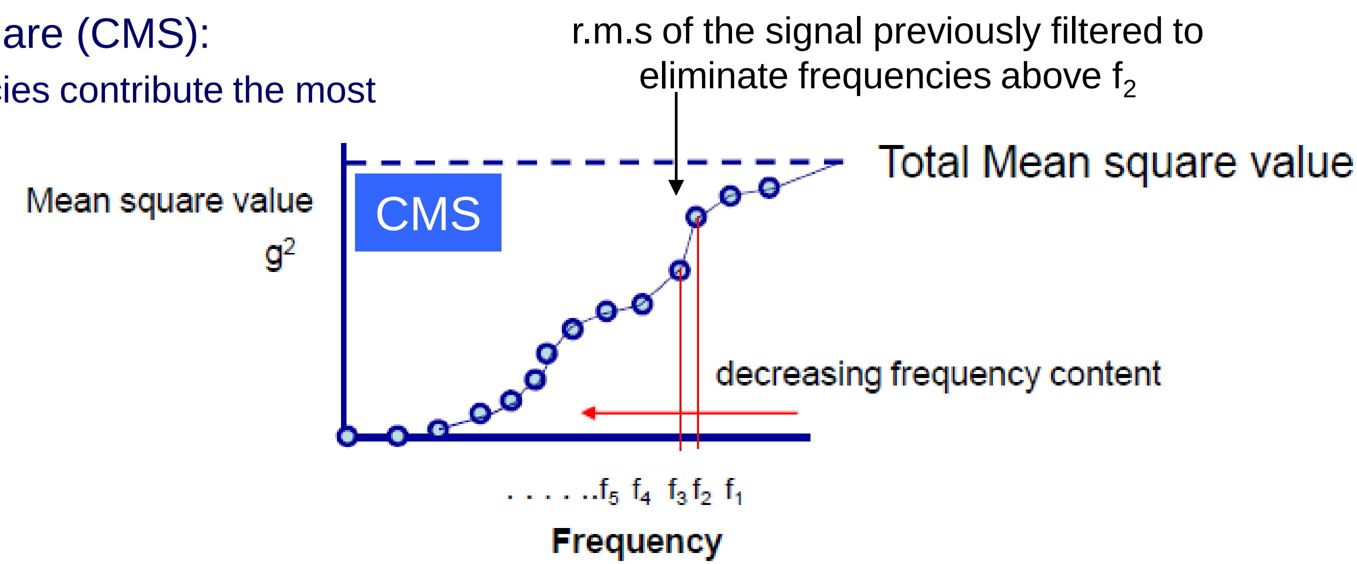


- Square the signal to get a non-zero mean
  - Non-zero mean value = mean square



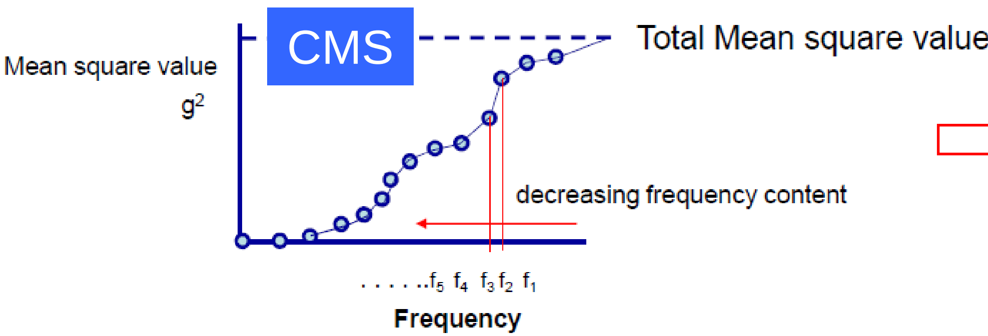
- It can be shown that the “Square Root of the Mean Square” (RMS) value is:
  - Equal to the standard deviation σ of a Normal Distribution → value that has 68.3% chance of occurring
  - “3 σ” gives a probability of 99.73% chance of occurring

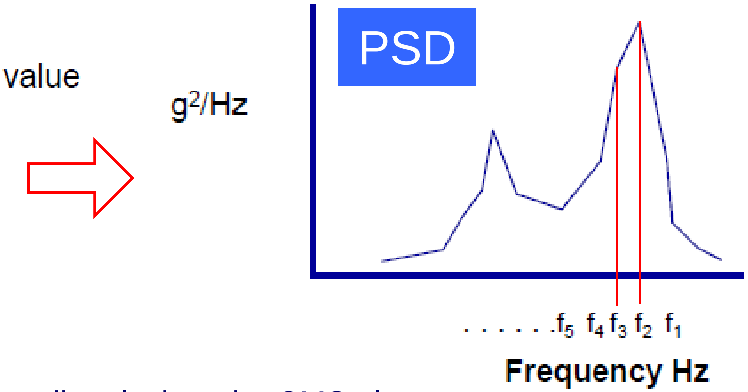
- Cumulative Mean Square (CMS):
  - Show which frequencies contribute the most



RANDOM SIGNALS
POWER SPECTRAL DENSITY "PSD"

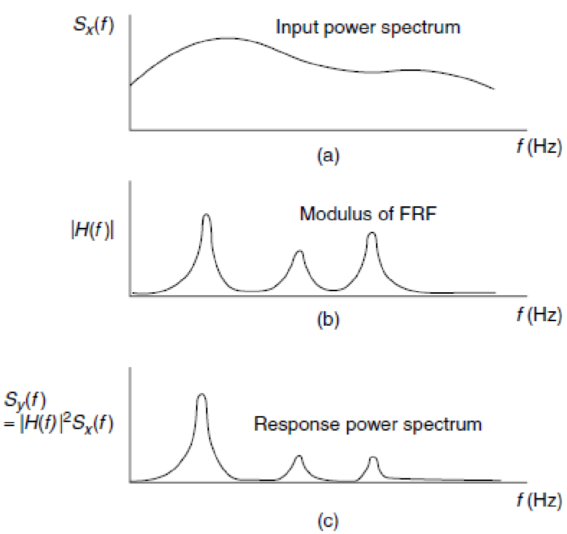
- Now take the gradient of the CMS:





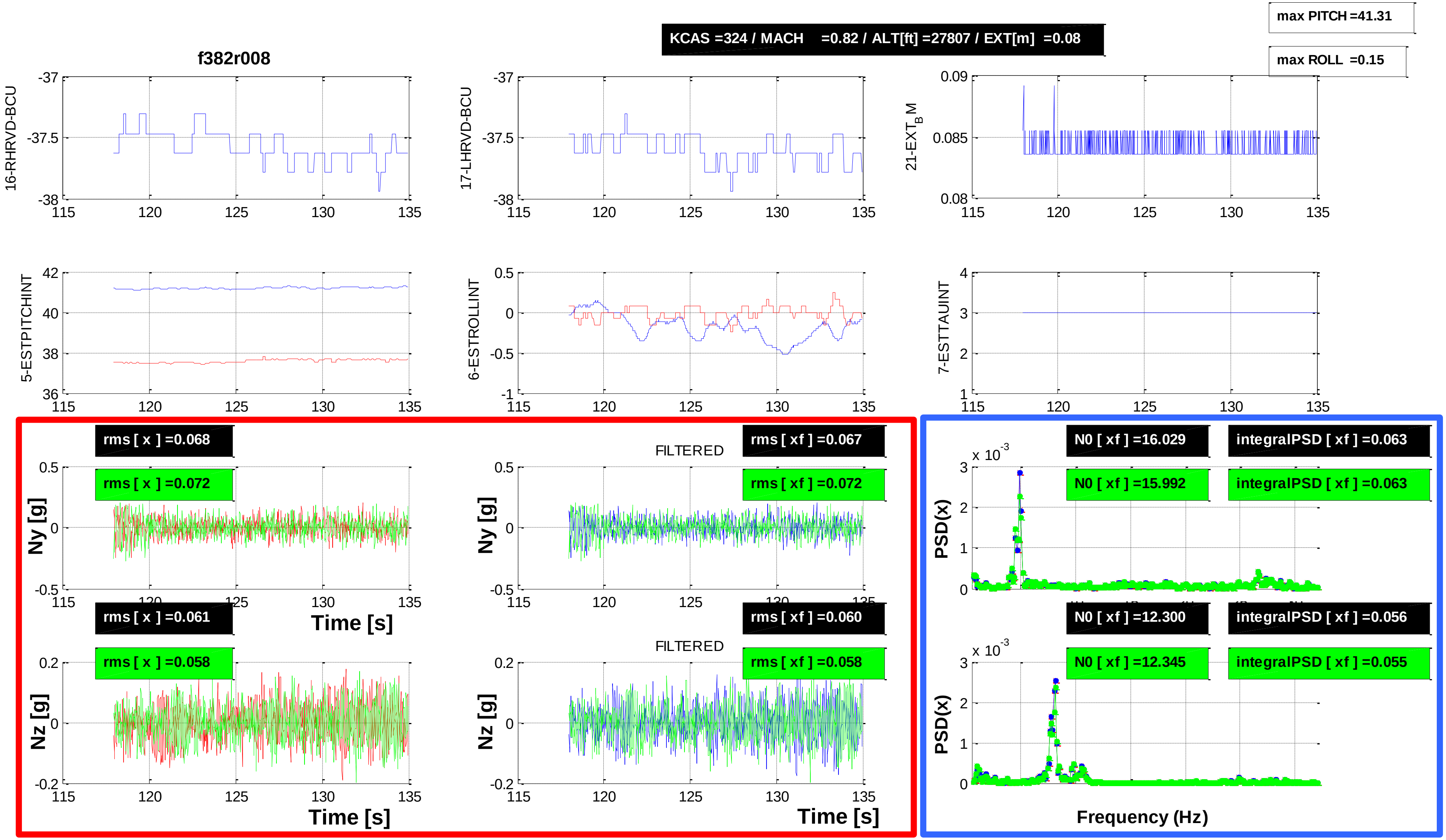
  - ► The PSD shows the frequency content of the signal more directly than the CMS plot
  - ► **The square root of the area under the PSD curve is the RMS !**

- Key point on using PSD in IN-OUT random processes :
  - ► "OUTPUT" PSD is related to the "INPUT" PSD thru Frequency Response Function H(f)



$$S_y(f) = |H(f)|^2 \cdot S_x(f)$$

EXAMPLE OF ANALYSIS OF RANDOM SIGNALS



RANDOM SIGNALS: Ny and Nz accelerations measured at a location in the airframe

PSD analysis of the time-histories

“GAUSSIAN” OR “NORMAL” DISTRIBUTION
VALUES WITH PROBABILITY 1/1000

$$\Phi_I(\Omega) = \frac{L}{\pi}\frac{1+\frac{8}{3}(1.339\Omega L)^2}{[1+(1.339\Omega L)^2]^{11/6}}$$

PSD of a σ_w =1 rms Random Gust

$$\bar{A}_i = \left[\int_0^\infty |h_i(\Omega)|^2 \;\phi_I(\Omega)d\Omega\right]^{1/2}$$

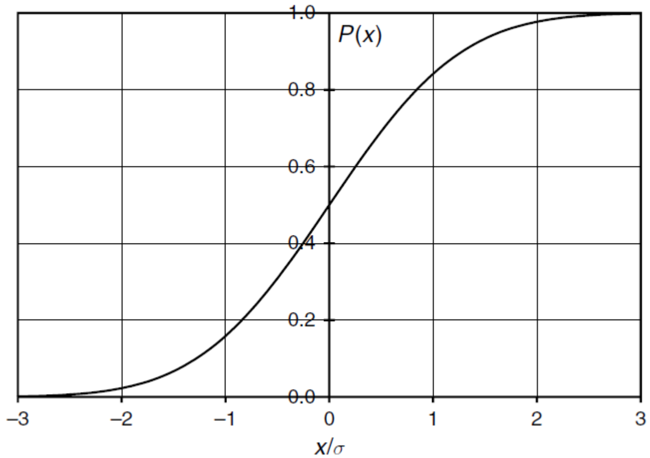
rms of a magnitude “i” for a turbulence with σ_w=1

$$P_{Li} = P_{(1-g)i} \pm P_{Ii} = P_{(1-g)i} \pm U_\sigma \bar{A}_i$$

It should be noted that the reference gust velocity is comprised of two components, a root-mean-square (RMS) gust intensity and a peak to RMS ratio.

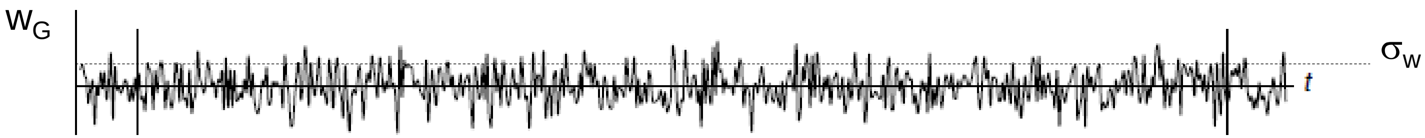
$$U_\sigma \approx 3 \cdot \sigma_w$$

PROBABILITY OF A “GAUSSIAN” O “NORMAL” DISTRIBUTION



Probability, $P(x)$, of a Gaussian random waveform lying between $-\infty$ and x.

x/σ	$P(x)$
$-\infty$	0
−3.1	0.0010
−3.0	0.0013
−2.5	0.0062
−2.0	0.0228
−1.5	0.0668
−1.0	0.1587
−0.5	0.3085
0	0.5000
0.5	0.6915
1.0	0.8413
1.5	0.9332
2.0	0.9772
2.5	0.9938
3.0	0.9987
3.1	0.9990
∞	1



$$\Omega = \frac{\omega}{U_\infty} = \frac{2\pi f}{U_\infty}$$

L = Scale of Turbulence = 2500

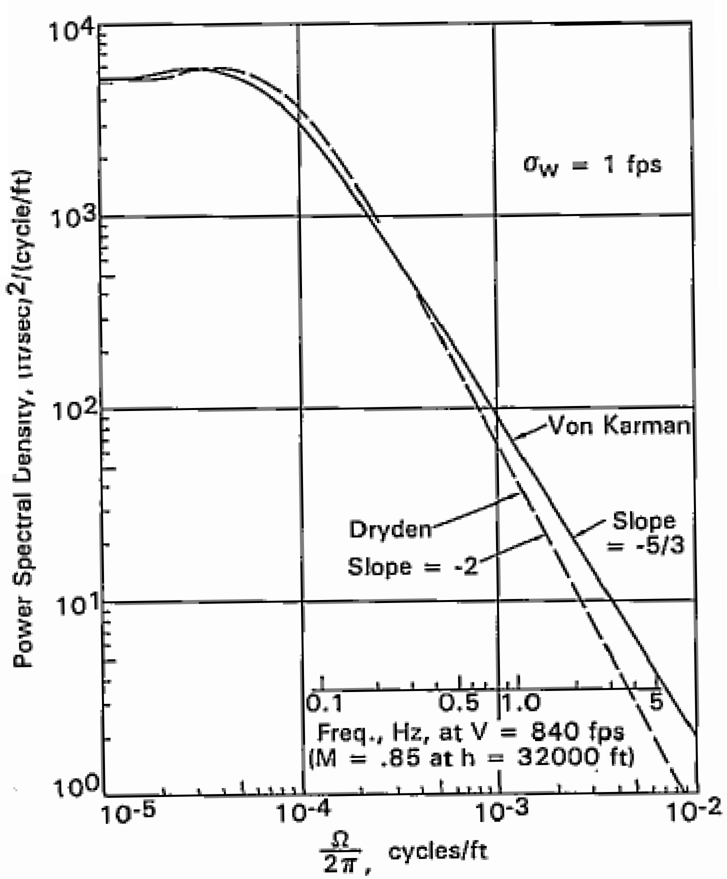
U_∞ = Flight Speed

Von Karman Model

$$\Phi(f) = \sigma_w^2 \frac{2L}{U_\infty} \frac{1 + \frac{8}{3}(1{,}339 L\Omega)^2}{\left[1 + (1{,}339 L\Omega)^2\right]^{\frac{11}{6}}}$$

$$\Phi\left(\frac{\Omega}{2\pi}\right) = \sigma_w^2 2L \frac{1 + \frac{8}{3}(1{,}339 L\Omega)^2}{\left[1 + (1{,}339 L\Omega)^2\right]^{\frac{11}{6}}}$$

$$\Phi(\omega) = \sigma_w^2 \frac{L}{\pi U_\infty} \frac{1 + \frac{8}{3}(1{,}339 L\Omega)^2}{\left[1 + (1{,}339 L\Omega)^2\right]^{\frac{11}{6}}}$$



Dryden Model

$$\Phi(f) = \sigma_w^2 \frac{2L}{U_\infty} \frac{1 + 3L^2\Omega^2}{(1 + L^2\Omega^2)^2}$$

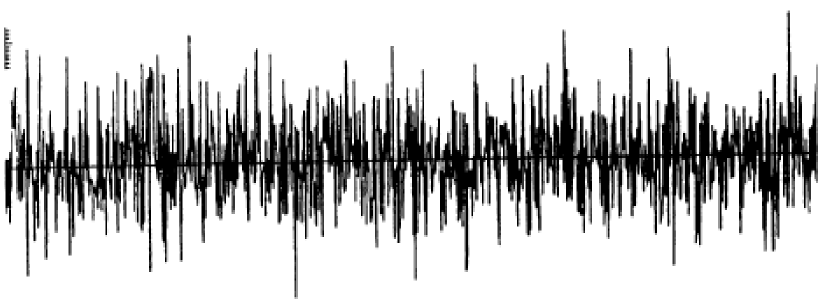
$$\Phi\left(\frac{\Omega}{2\pi}\right) = \sigma_w^2 2L \frac{1 + 3L^2\Omega^2}{(1 + L^2\Omega^2)^2}$$

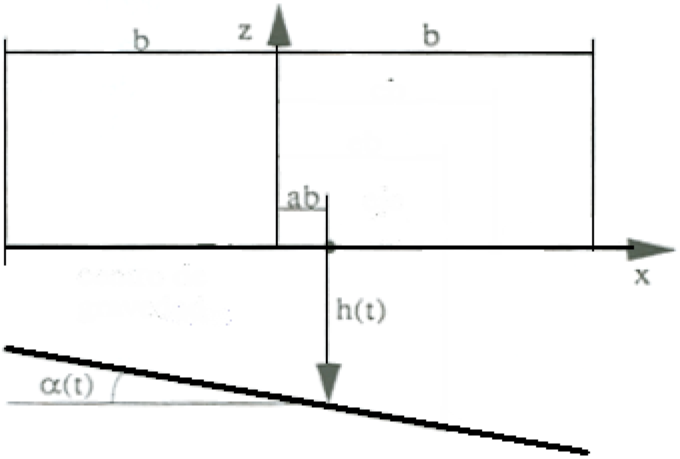
$$\Phi(\omega) = \sigma_w^2 \frac{L}{\pi U_\infty} \frac{1 + 3L^2\Omega^2}{(1 + L^2\Omega^2)^2}$$

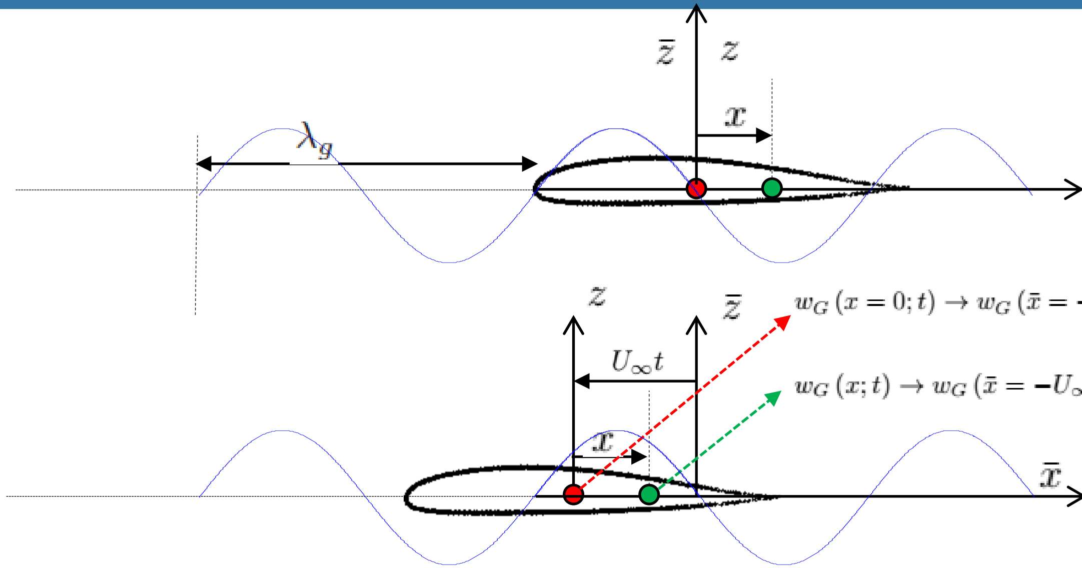
NOW… LET'S FIX CONCEPTS WITH OUR TYPICAL SECTION

For the sake of simplicity, let's assume:

- ► Incompressible flow (Theodorsen's theory remains applicable)
- ► Only heave motion **h(t)**
- ► No stiffness ($K_h = 0$) and zero structural damping ($g_h = 0$)







$$w_G(\bar{x}) = w_0 \sin\left(\frac{2\pi}{\lambda_g}\bar{x}\right)$$

$$w_G(x=0;t) \to w_G(\bar{x} = -U_\infty t) = w_0 \sin\left(-\frac{2\pi}{\lambda}U_\infty t\right) = w_0 \sin(-\omega t)$$

$$w_G(x;t) \to w_G(\bar{x} = -U_\infty t + x) = w_0 \sin\left(-\omega t + \omega\frac{x}{U_\infty}\right) \to w_G = w_0 e^{-i\omega t} e^{+i\omega\frac{x}{U_\infty}}$$

$$w_G(x;t) = w_0 e^{i\omega x/U_\infty} e^{-i\omega t} = \bar{w}_G(x)\, e^{-i\omega t} \Rightarrow \bar{w}_G(x) = w_0 e^{i\omega x/U_\infty} = w_0 e^{ikx/b}$$

THEODORSEN'S FORMULATION
(see presentation #19)

$$\frac{\bar{w}(x)}{U_\infty} = -\frac{\bar{w}_G}{U_\infty} = -\frac{w_0}{U_\infty} e^{ikx/b}$$

$$\bar{\gamma}_a(x;k) = \frac{2}{\pi}\sqrt{\frac{1-x}{1+x}}\left[\int_{-1}^{1}\sqrt{\frac{1+\xi}{1-\xi}}\,\frac{\overline{w}(\xi,0;k)}{x-\xi}d\xi + \frac{ik\overline{\Gamma}e^{ik}}{2}\int_{1}^{\infty}\frac{e^{-ik\lambda}}{x-\lambda}\sqrt{\frac{\lambda+1}{\lambda-1}}d\lambda\right]$$

$$\overline{\Gamma} = \frac{4\int_{-1}^{1}\sqrt{\frac{1+\xi}{1-\xi}}w(\xi,0,k)d\xi}{\pi e^{ik}ik\left[H_1^{(2)}(k) + iH_0^{(2)}(k)\right]}$$

$$\Rightarrow \Delta\bar{C}_{p1} = \frac{2}{U_\infty^2}\left(U_\infty\bar{\gamma}_a(x;k) + i\omega\int_{-b}^{x}\bar{\gamma}_a(x;k)dx\right) \Rightarrow L_G$$

$$L_G = 2\pi\rho_\infty U_\infty b\bar{w}_G \left\{C(k)\left[J_0(k) - iJ_1(k)\right] + iJ_1(k)\right\} e^{iks} = 2\pi\rho_\infty U_\infty bS(k)\bar{w}_G$$

$$L_M = \pi\rho_\infty U_\infty^2 \ddot{h}(s) + 2\pi\rho_\infty U_\infty^2 C(k)\dot{h}(s)$$

$$M\frac{U_\infty^2}{b^2}\ddot{h}(s) = -2\pi\rho_\infty U_\infty bS(k)\bar{w}_G - \pi\rho_\infty U_\infty^2\ddot{h}(s) - 2\pi\rho_\infty U_\infty^2 C(k)\dot{h}(s)$$

$$\frac{-\omega^2\frac{\bar{h}}{b}}{\bar{w}_G} = -\frac{U_\infty}{2b^2}\frac{S(k)}{\lambda + \frac{1}{4} - i\frac{1}{2}\frac{C(k)}{k}} = H(ik)$$

$$\lambda = \frac{M}{4\pi\rho_\infty b^2}$$

$$\Psi(\omega) = |H(\omega)|^2 \Phi(\omega) = \frac{U_\infty^2}{b^2} \frac{4k^2 |S(k)|^2}{|-(4\lambda+1)k + 2iC(k)|^2} \sigma_w^2 \frac{L}{\pi U_\infty} \frac{1+3\left(\frac{\omega L}{U_\infty}\right)}{\left[1+\left(\frac{\omega L}{U_\infty}\right)^2\right]^2}$$

$$kS(k) \approx \frac{1}{1+2\pi k} = \frac{\frac{\omega L}{U_\infty}\frac{b}{L}}{1+2\pi\frac{\omega L}{U_\infty}\frac{b}{L}}$$

$$|-(4\lambda+1)k + 2iC(k)|^2 \approx 4 + k^2(2\lambda+1)^2 = 4 + \left(\frac{\omega L}{U_\infty}\right)^2 \left(\frac{b}{L}\right)^2 (2\lambda+1)^2$$

$b / L \rightarrow 0$ Scale of turbulence >> characteristic length of the vehicle $\rightarrow d^2h / dt^2 \approx 0$

$b / L \rightarrow \infty$ Scale of turbulence << characteristic length of the vehicle $\rightarrow d^2h / dt^2 \approx 0$

$(b / L)_{crit}$ Scale of turbulence associated to maximum vertical acceleration

PROBLEMA 1

Determinar el factor de carga de un ala rígida pero libre de desplazarse verticalmente cuando encuentra una ráfaga de intensidad $w_G(s) = w_0 \cos(\Omega s)$, siendo $s = U_\infty t/b$ el tiempo adimensional, b la semicuerda del ala y las funciones de Wagner y Küssner aproximándose por:

$$\Phi(s) = 1 - 0{,}165e^{-0{,}0455s} - 0{,}335e^{-0{,}300s}$$

$$\Psi(s) = 1 - 0{,}500e^{-0{,}130s} - 0{,}500e^{-s}$$

Expresar el resultado en función de parámetros adimensionales.

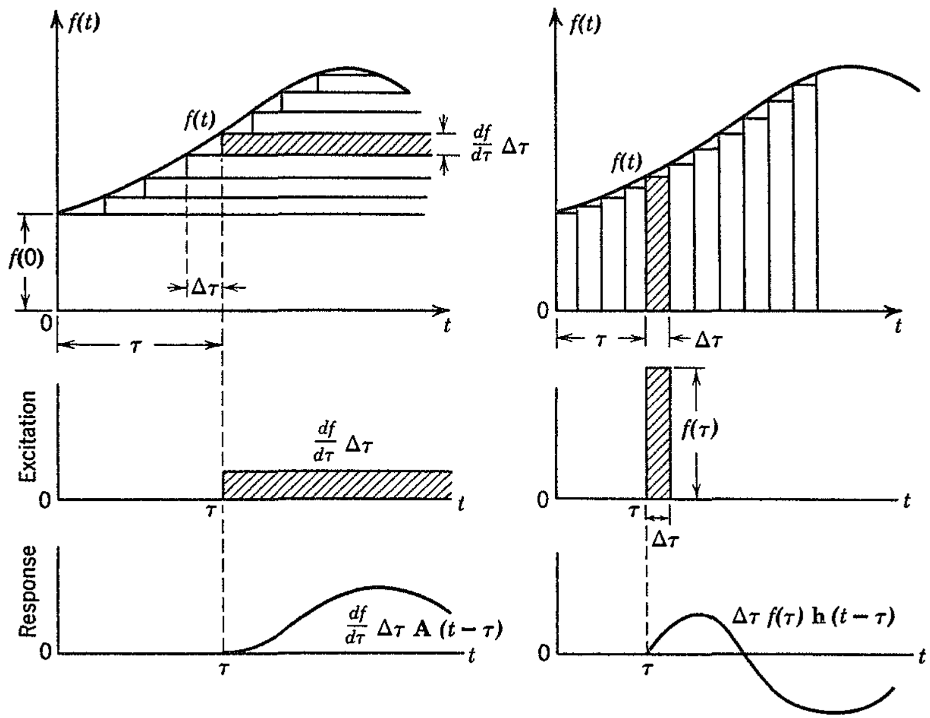
PROBLEMA 2

Dentro de la validez de la teoría aerodinámica bidimensional, incompresible y no estacionaria, se desea estudiar la respuesta de un puente frente a ráfagas. Para ello se considera una sección característica de anchura $2b$ y masa por unidad de envergadura M en presencia de un viento horizontal U_∞. Repentinamente, y durante un tiempo t_1, aparece una velocidad vertical uniforme $w_0 << U_\infty$. Teniendo en cuenta que tanto el espesor como el desplazamiento vertical de la sección característica son pequeños frente a su anchura, calcular la evolución temporal dela velocidad vertical de la sección, considerando que no gira y que la rigidez a flexión de la sección analizada es $M\omega_h^2$.

Dibujar el resultado anterior cuando la masa vale $M = 4383\ Kg.m^{-1}$, la anchura del puente es 4 [m], la rigidez a flexión es nula y la duracción de la ráfaga es infinita.

DUHAMEL'S INTEGRAL
PRINCIPLE OF SUPERPOSITION

x(t) response of the physical system to the input **f(t)**



A(t)

"Indicial Admittance" = response to unit-step function

h(t)

response to unit-step function

$$x(t) = f(0)\,A(t) + \int_0^t \frac{df}{dt}(\tau)\,A(t-\tau)\,d\tau$$

$$x(t) = \int_0^t f(\tau)\,h(t-\tau)\,d\tau$$

$$h(t) = A(0)\,\delta(t) + \frac{dA}{dt}(t)$$

LAPLACE TRANSFORM
CONVOLUTION THEOREM

$$\mathcal{L}\left\{f\left(t\right)\right\} = \int_0^\infty f(t)e^{-st}dt$$

$$\mathcal{L}^{-1}\{F(s)G(s)\} = (f * g)(t)$$

$$(f * g)(t) = \int_{-\infty}^{\infty} f(t-x)g(x)\mathrm{d}x$$

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